

# Planning for $LTL_f$ Goals in Presence of Uncertainty on the Initial State

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ERC Advanced Grant

WhiteMech:

White-box Self Programming Mechanisms



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# Motivation

We are interested in **Planning for Temporally Extended Goals**.

This setting has been extensively studied in the literature, e.g.:

- use of Temporally Extended Goals on Infinite Traces [De Giacomo & Vardi 1999]
  - they handle uncertainty on initial state, but only for the infinite traces setting
- use of Temporally Extended Goals on Finite Traces
  - deterministic planning [Baier & McIlraith 2006] and non-deterministic planning [DeGiacomo & Rubin 2018]
  - they handle temporal goals on finite traces, but without uncertainty on initial state

***Goal:*** *we extend the work of [De Giacomo, Vardi 1999] in the finite traces setting (with LTLf goals)*

# Linear Temporal Logic on Finite Traces

$LTL_f$ : the language

$$\varphi ::= A \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \mathcal{U} \varphi_2$$

- $A$ : atomic propositions
- $\neg\varphi, \varphi_1 \wedge \varphi_2$ : boolean connectives
- $\bigcirc\varphi$ : “next step exists and at next step (of the trace)  $\varphi$  holds”
- $\varphi_1 \mathcal{U} \varphi_2$ : “eventually  $\varphi_2$  holds, and  $\varphi_1$  holds until  $\varphi_2$  does”
- $\bullet\varphi \doteq \neg\bigcirc\neg\varphi$ : “if next step exists then at next step  $\varphi$  holds” (*weak next*)
- $\diamond\varphi \doteq \text{true} \mathcal{U} \varphi$ : “ $\varphi$  will eventually hold”
- $\Box\varphi \doteq \neg\diamond\neg\varphi$ : “from current till last instant  $\varphi$  will always hold”
- $Last \doteq \neg\bigcirc\text{true}$ : denotes last instant of trace.

**Properties:** expressibility (FOL over finite sequences or Star-free RE), reasoning (satisfiability, validity, entailment PSPACE-complete), model checking (linear on TS, PSPACE-complete on formula)

# Non Deterministic Finite State Automata

An NFA is a tuple  $\mathcal{A} = (\Sigma, Q, Q_0, \delta, F)$ :

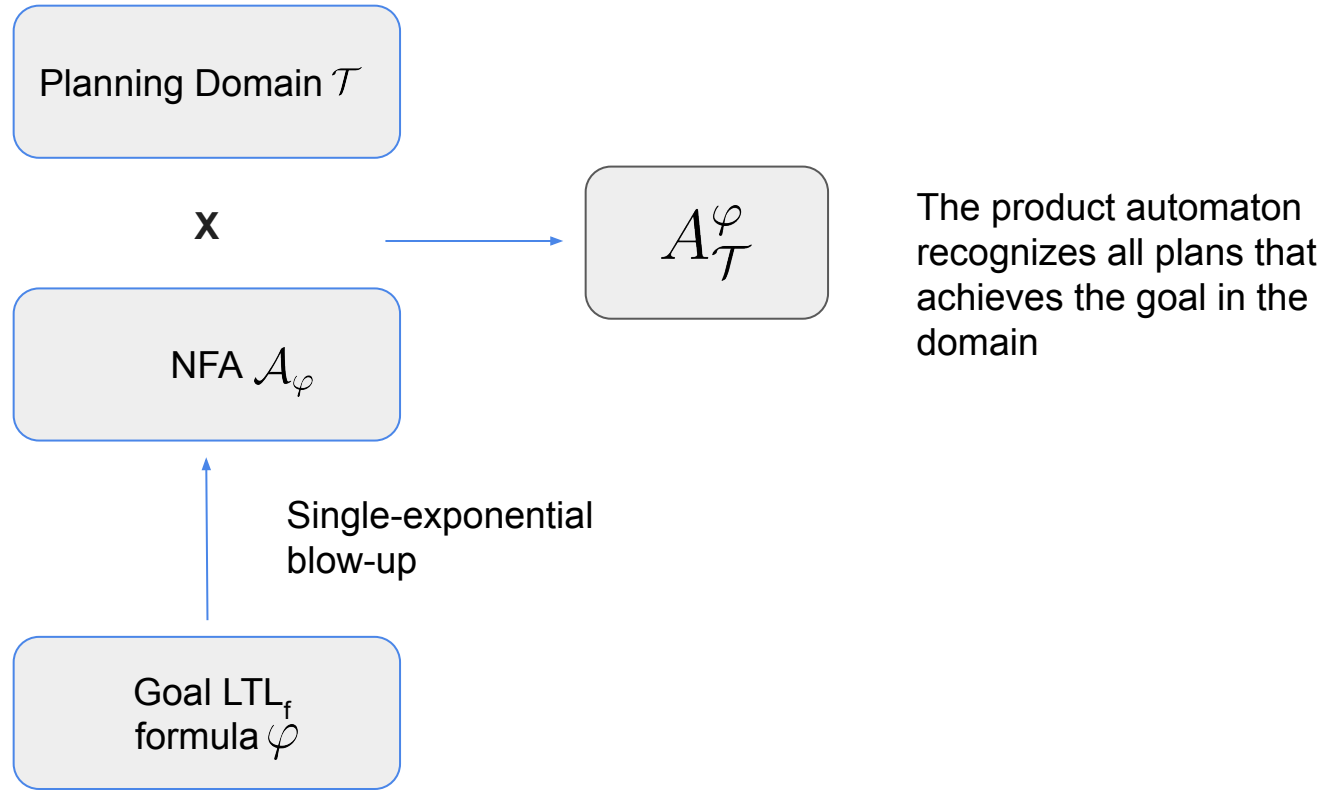
- $\Sigma$  is the alphabet;
- $Q$  is a finite set of states;
- $Q_0 \subseteq Q$  is the set of initial states;
- $\delta = Q \times \Sigma \rightarrow 2^Q$  is the nondeterministic transition relation;
- $F \subseteq Q$  is the set of accepting states

# Planning Domain

A planning domain is transition system  $\mathcal{T} = (S, S_0, Act, R, Obs, \pi)$

- $S$  is the finite set of possible states;
- $S_0 \subseteq S$  is the finite set of possible initial states;
- $Act$  is the set of possible actions;
- $R : S \times Act \rightarrow S$  is the transition function that given a state and an action returns the next state;
- $Obs$  is the finite set of possible observations
- $\pi : S \rightarrow Obs$  is the observability function

# Planning for $LTL_f$ goals: the idea



# Standard Planning

Complete information on the initial state: initial state is a singleton  $\{s_0\} \subseteq S$ , set of observations coincides with the set of states  $Ob_S = S$

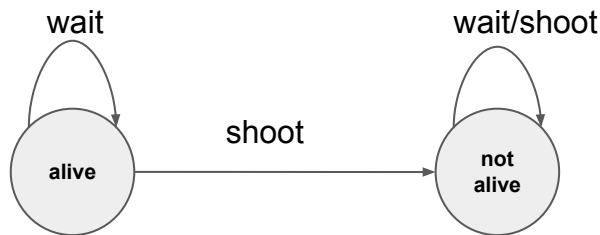
To synthesize a plan compute NFA  $\mathcal{A}_{\mathcal{T}}^{\varphi} = (Act, \mathcal{S}_T, \mathcal{S}_{T_0}, \delta_T, F_T)$  :

- $\mathcal{S}_T = Q \times S$
- $\mathcal{S}_{T_0} = Q_0 \times \{s_0\}$
- $(q_j, s_j) \in \delta_T((q_i, s_i), a)$  iff  $s_j = R(s_i, a)$  and  $q_j \in \delta(q_i, \pi(s_i))$
- $F_T = F \times S$

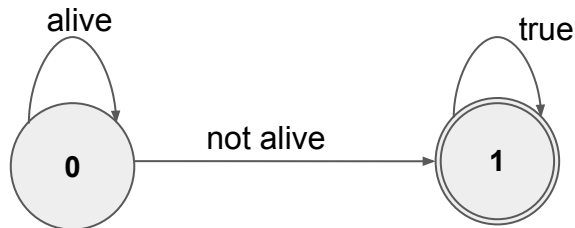
**Theorem:** A plan  $p$  for  $\mathcal{T}$  realizing the specification  $\mathcal{A}_{\varphi}$  exists iff  $L(\mathcal{A}_{\mathcal{T}}^{\varphi}) \neq \emptyset$ .

# Standard Planning-Yale Shooting Example

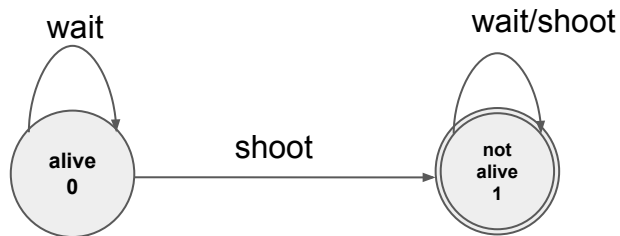
Yale Shooting  
Domain  $\mathcal{T}$



$\mathcal{A}_{\diamond \neg alive}$



$\mathcal{A}_{\mathcal{T}}^{\diamond \neg alive}$





# Conformant Planning

Incomplete information on the initial state and no observability on the states:  $S_0 = \{s_{00}, \dots, s_{0k-1}\}$  for  $k > 1$

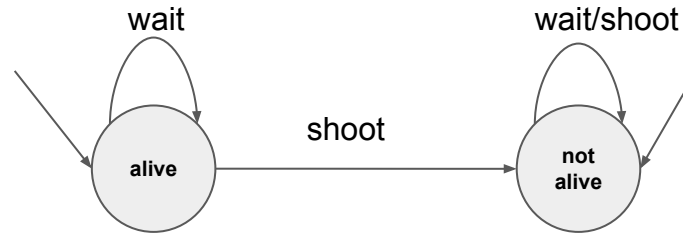
To synthesize a plan compute NFA  $\mathcal{A}_{\mathcal{T}}^{\varphi} = (Act, \mathcal{S}_T, \mathcal{S}_{T_0}, \delta_T, F_T)$ :

- $\mathcal{S}_T = Q^k \times S^k$
- $\mathcal{S}_{T_0} = Q_0^k \times \{(s_{00}, \dots, s_{0k-1})\}$
- $(\vec{q}_j, \vec{s}_j) \in \delta_T((\vec{q}_i, \vec{s}_i), a)$  iff  $s_{jh} = R(s_{ih}, a)$  and  $q_{jh} \in \delta(q_{ih}, \pi(s_{ih}))$   
for  $h = 0, \dots, k-1$
- $F_T = F^k \times S^k$

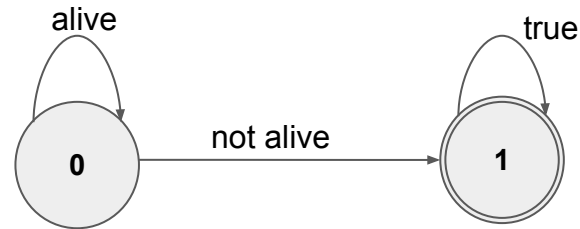
**Theorem:** A plan  $p$  for  $\mathcal{T}$  realizing the specification  $\mathcal{A}_{\varphi}$  exists iff  $L(\mathcal{A}_{\mathcal{T}}^{\varphi}) \neq \emptyset$ .

# Conformant Planning-Yale Shooting Example

Yale Shooting  
Domain  $\mathcal{T}$



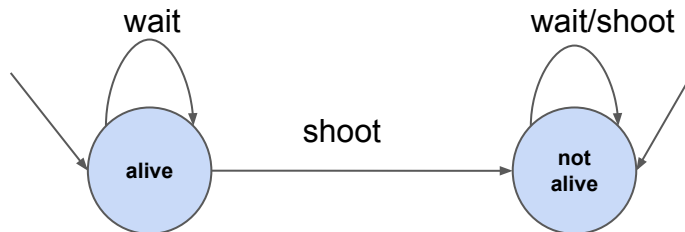
$\mathcal{A}_{\diamond \neg \text{alive}}$



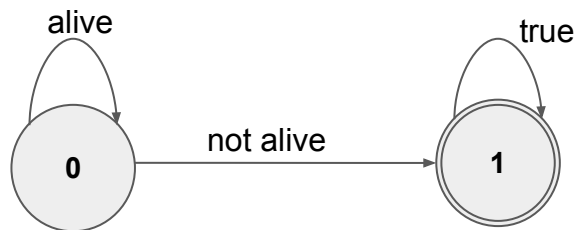
# Conformant Planning-Yale Shooting Example

Possible initial states

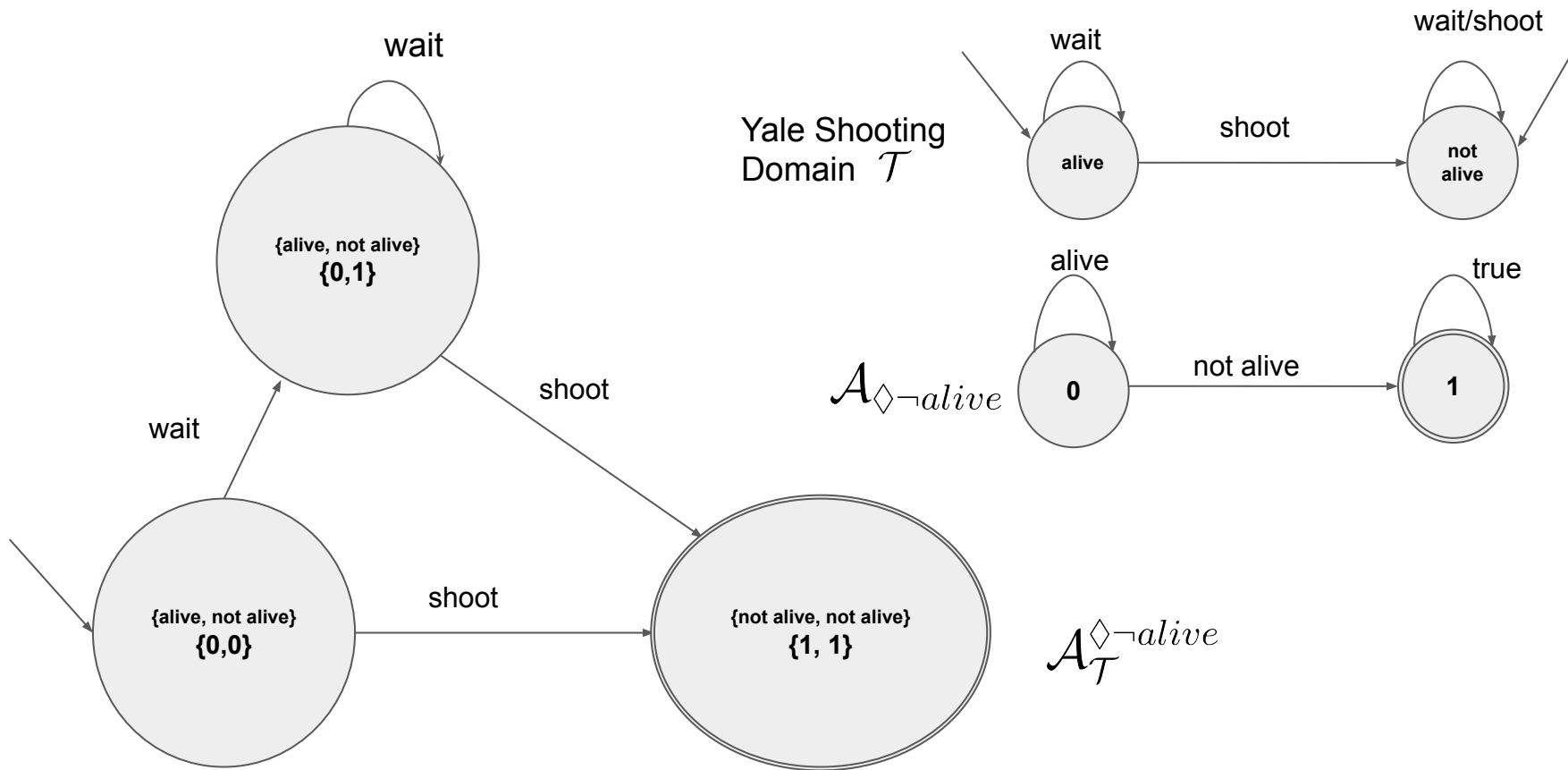
Yale Shooting  
Domain  $\mathcal{T}$



$\mathcal{A}_{\diamond \neg \text{alive}}$



# Conformant Planning-Yale Shooting Example



# Contingent Planning

Incomplete information on the initial state and observability on the states

To synthesize a plan compute NFA  $A_{\mathcal{T}}^{\varphi} = (Act^k, \mathcal{S}_T, \mathcal{S}_{T0}, \delta_T, F_T)$  :

- $\mathcal{S}_T = Q^k \times S^k \times \mathcal{E}_k$
- $\mathcal{S}_{T0} = Q_0^k \times \{(s_{00}, \dots, s_{0k-1})\} \times \equiv_0$  where  $i \equiv_0 j$  iff  $s_{0i} = s_{0j}$
- $(\vec{q}_j, \vec{s}_j, \equiv') \in \delta_T((\vec{q}_i, \vec{s}_i), \vec{a}, \equiv)$  iff  $s_{jh} = R(s_{ih}, a_h)$  and

if  $l \equiv m$  then  $a_l = a_m, l \equiv' m$  iff  $l \equiv m$  and  $\pi(s_{jl}) = \pi(s_{jm})$

- $F_T = F^k \times S^k \times \mathcal{E}_k$

**Theorem:** A plan  $p$  for  $\mathcal{T}$  realizing the specification  $\mathcal{A}_{\varphi}$  exists iff  $L(\mathcal{A}_{\mathcal{T}}^{\varphi}) \neq \emptyset$ .

# Conclusion

## Contributions:

- ❑ Extension of the work of [De Giacomo, Vardi 1999] in the finite traces setting with LTLf goals
- ❑ The techniques introduced here can be extended to work for a wide range of transition-based formalisms
- ❑ The approach works for any temporal specification language that can be translated into automaton (e.g.  $LDL_f$ , PPLTL, PPLDL etc.)

## Future works:

- ❑ Investigate the complexity of the planning problems
- ❑ Provide implementations of the presented techniques
- ❑ Find interesting use-cases and applications