LINEAR TEMPORAL LOGIC MODULO THEORIES OVER FINITE TRACES

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Who are we?

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Formal verification,
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Formal verification,
data-aware systems,
business process modeling.

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Formal verification,
temporal planning,
temporal logics.

Linear Temporal Logic

Linear Temporal Logic (LTL) is the most common formalism to specify temporal properties in **formal verification** and **artificial intelligence**.

Limits of propositional logics

The **propositional** nature of LTL and similar logics limits them to **finite-state** systems.

However, many scenarios are difficult or impossible to abstract finitely:

- systems involving **arithmetics**
- systems involving complex and unbounded **data structures**
- systems involving **relational databases**

LTL modulo theories

For this reason, we introduced **LTLf modulo theories** ($\mathsf{LTL}^{\mathsf{MT}}$) [GGG22]:

- first-order extension of LTLf
- propositions are replaced by **first-order sentences** over arbitrary theories, *à la* SMT
- (semi-)decision procedures based on off-the-shelf **SMT solvers**

LTL modulo theories

Many first-order extensions of LTL have been studied, however:

- many first-order temporal logics have been extensively studied from **theoretical** perspectives but without any practical development (see, *e.g.* [Kon+04])
- others led to practically applicable approaches but support quite **ad-hoc** syntax and semantics (see, *e.g.* [Cim+20])

Our approach is at the same time **theoretically** well-grounded, **general**, and **practically** oriented.

LTLf^{MT} is supported by our **BLACK**[1] temporal reasoning framework:[2]

- a software library and tool for **temporal reasoning** in linear-time logics
- supports LTL/LTLf and LTLf^{MT} in many flavors
- playground for many of our research directions

[1] Bounded LTL sAtisfiability ChecKer
[2] https://www.black-sat.org

Data-aware systems

Data-aware systems

Systems that involve the processing and manipulation of **data** taken from an **infinite** domain.

Examples:

- (relational) database-driven systems
- systems involving complex data-structures
- systems involving arithmetics
- any combination of the above!

Data-aware systems are **infinite-state**, leading very easily to **undecidability** of verification, model-checking, satisfiability etc ...

But they are still worth studying!

LTLf modulo theories

LTLf$^{\text{MT}}$ is our take at the verification of **infinite-state** systems.

LTLf$^{\text{MT}}$ extends LTLf by replacing **propositions** with **first-order sentences**.

- symbols can be uninterpreted, or interpreted by arbitrary first-order theories
  - *e.g.*, $+$, $<$ interpreted as integer sum/comparison
- constants, relational/function symbols, *etc.* can be both **rigid** or **non-rigid**
- interpreted over **finite-traces**

Examples

$$\mathsf{G}(x = 2y) \qquad (x < y)\,\mathsf{U}\,(y = 0) \qquad \mathsf{G}(x > 5) \wedge \mathsf{F}(x = 0)$$

$$\mathsf{G}(\exists y(x = 2y))$$

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Examples

$$x = 0 \land ((\bigcirc x = x + 1) \, \mathsf{U} \, x = 42)$$

$$y = 1 \land \mathsf{G}(\widetilde{\bigcirc} \, y = y + 1 \land x = 2y)$$

$$p(0) \land \mathsf{G}\forall x(p(x) \to \widetilde{\mathsf{X}} p(x + 1)) \land \mathsf{F} p(42)$$

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Where's the catch?

LTLf^{MT} is clearly **undecidable**, but:

- over **decidable** first-order theories/fragments, it is **semi-decidable**
- our **semi-decision** procedure always answers **yes** for satisfiable formulas, **may** not terminate for unsatisfiable ones (but sometimes does)
- decidable theories and first-order fragments abound, *e.g.*:
  - linear integer/real **arithmetic** (LIA/LRA)
  - quantifier-free equality and **uninterpreted** functions (QF_EUF)
  - arrays, fixed-size bitvectors, algebraic data types, **floating-point** numbers, *etc.*
  - **effectively propositional** (EPR) logic: $\exists^*\forall^*\phi$
  - **two-variables** first-order logic (FO^2)

Why finite traces?

In propositional LTLf, finite traces makes everything simpler.

- *e.g.*, NFAs vs Büchi automata

However, complexities remain the same.

In the first-order world, this is not the case!

- LTLf$^{\mathsf{MT}}$ is semi-decidable for decidable first-order theories
- instead, for many decidable theories, LTL$^{\mathsf{MT}}$ is **not even semi-decidable**!

Why?

- the difference between **tiling** and recurrent tiling

So the finite-traces semantics is the only one giving us any **hope** of solving anything.

How to solve LTLf modulo theories

How do we test satisfiability of $\mathrm{LTLf}^{\mathrm{MT}}$ formulas?

- an **iterative** procedure tests the existence of models of length up to $k \geqslant 0$, for increasing values of k
- given an $\mathrm{LTLf}^{\mathrm{MT}}$ formula ϕ and a k, we build a **purely first-order formula** $\langle\phi\rangle_k$ that is **satisfiable** if and only if there is a model for ϕ of length at most k
- $\langle\phi\rangle_k$ is given to an off-the-shelf SMT solver

Some experiments

That's cool, but **does it work**?

- everything here is **undecidable**

Some experiments

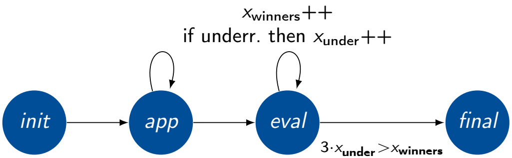
That's cool, but **does it work**?

- everything here is **undecidable**
- but. . .

Some early experiments

Test setting:

- simulation of a company hiring process
- nondeterministic transitions:
  - dependent on arithmetic constraints
  - acting on unbounded relational data
- minimal length of the counterexamples dependent over scalable parameter N
- two modelings of the same system:
  - P_1 employs arithmetic constraints
  - P_2 avoids arithmetics, simulates constraints by other means
- two different properties for each variant



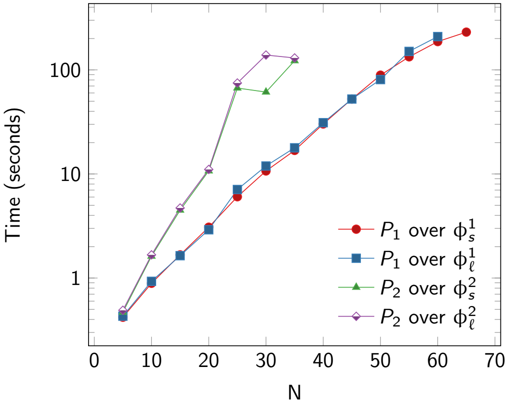
$$\phi_s^1 \equiv \mathsf{G}(x_{state} = \mathit{final} \rightarrow 2x_{\text{under}} > x_{\text{winners}})$$

$$\phi_\ell^1 \equiv \mathsf{G}\begin{pmatrix} x_{state} = \mathit{app} \rightarrow \\ F(x_{\text{state}} = \mathit{final} \wedge 2x_{\text{under}} > x_{\text{winners}}) \end{pmatrix}$$

Some early experiments

Results:

- 5 minutes timeout reached at $N = 70$
- **exponential** growth
  - but could be much worse, the problem is **undecidable**!
- liveness property not harder than the safety one
- system with **explicit arithmetics** faster to verify
- everything implemented in **BLACK**



Future directions

Where to go from now?

- find **decidable** $\mathsf{LTL}^{\mathsf{MT}}$ and $\mathsf{LTLf}^{\mathsf{MT}}$ fragments
- find more **efficient** $\mathsf{LTLf}^{\mathsf{MT}}$ fragments (not necessarily decidable)
- reactive **synthesis** for $\mathsf{LTLf}^{\mathsf{MT}}$ objectives
- theoretical properties of $\mathsf{LTLf}^{\mathsf{MT}}$
- **automata modulo theories**

THANK YOU

REFERENCES

[Cim+20] Alessandro Cimatti, Alberto Griggio, Enrico Magnago, Marco Roveri, and Stefano Tonetta. “SMT-based satisfiability of first-order LTL with event freezing functions and metric operators.” In: *Inf. Comput.* 272 (2020), p. 104502. DOI: 10.1016/j.ic.2019.104502.

[GGG22] Luca Geatti, Alessandro Gianola, and Nicola Gigante. “Linear Temporal Logic Modulo Theories over Finite Traces.” In: *Proceedings of the Thirty-First International Joint Conference on Artificial Intelligence*. ijcai.org, 2022, pp. 2641–2647. DOI: 10.24963/ijcai.2022/366.

[Kon+04] Roman Kontchakov, Carsten Lutz, Frank Wolter, and Michael Zakharyaschev. “Temporalising Tableaux.” In: *Stud Logica* 76.1 (2004), pp. 91–134. DOI: 10.1023/B:STUD.0000027468.28935.6d.