

# Towards Dynamic, Metric and Temporal Answer Set Programming over Linear Finite Traces

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- Origin Temporal logic of Here-and-There (Cabalar and Pérez, 2007) over infinite traces

# The logic of Here-and-There

## ■ Origin

Three valued logic due to (Heyting, 1930; Gödel, 1932)

- HT is based on Kripke semantics for intuitionistic logic
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- Implication is a genuine connective, and negation is defined in terms of implication:  $\neg\varphi = \varphi \rightarrow \perp$

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 $(\mathbf{H}', \mathbf{T}), k \not\models \varphi$  or  $(\mathbf{H}', \mathbf{T}), k \models \psi$ , for all  $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$

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  - Something Dynamic  $(\mathbf{H}, \mathbf{T}), k \models [\rho] \varphi$  if  
 $(\mathbf{H}', \mathbf{T}), i \models \varphi$  for all  $i = 0..{\lambda}$  with  $(k, i) \in \parallel \rho \parallel^{(\mathbf{H}', \mathbf{T})}$ ,  
for all  $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$

# Pedestrian traffic light

Metric equilibrium logic

$$\Box(\text{red} \wedge \text{green} \rightarrow \perp) \quad (1)$$

$$\Box(\neg \text{green} \rightarrow \text{red}) \quad (2)$$

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- $\{(1), (2), (3)\} \models_{MEL} \Box(\text{red} \wedge \neg \text{green} \wedge \neg \text{push})$
- $\{(1), (2), (3), \circ_5 \text{push}\}$  has 14 metric equilibrium models of length 3
  - $T_0 = \{\text{red}\} \quad \tau(0) = 0$
  - $T_1 = \{\text{push}, \text{red}\} \quad \tau(1) = 5$
  - $T_2 = \{\text{green}\} \quad \tau(2) \in \{6, \dots, 19\}$

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- relies on finite traces
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- Example *“A robot cannot lift a box unless its capacity exceeds the box’s weight plus that of all held objects”*

```
:- lift(R,B), robot(R), box(B,W),  
   #sum { C : capacity(R,C);  
         -V,0 : 'holding(R,0,V) } < W.
```



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<sup>1</sup>Classical logic is obtained in ASP by adding choices; eg., ' $\{a\}.$ ' stands for ' $a \vee \neg a$ '.